APPROVAL CRITERIA FOR GCE AS AND A LEVEL FURTHER MATHEMATICS

JULY 2016

Contents

This is a **Regulatory Document** under **Condition B7** of the *Interim Standard Conditions of Recognition*[1]*: Compliance with Regulatory Documents.*

[1] http://qualificationswales.org/regulation/monitoring-awarding-bodies/?lang=en&

Introduction

This document sets out the approval criteria for AS and A Level Further Mathematics qualifications. These have been developed through stakeholder engagement and public consultation. They include the requirements that an awarding body must meet when developing the specification and assessment materials for the qualification.

The approval criteria in this document will come into effect from 18 July 2016.

Qualifications Wales will only approve qualifications that meet all of the requirements set out in this document together with those set out in the *GCE AS and A Level Qualification Approval Criteria*[2] and *Interim Standard Conditions of Recognition*[3]. In developing qualifications to meet these requirements awarding bodies must have regard to *Fair Access by Design*[4].

Where the requirements of the Subject Approval Criteria set out in this document differ from those prescribed in the *GCE AS and A Level Qualifications Approval Criteria* and the *Interim Standard Conditions of Recognition*, the requirements in this document will take precedence.

[2] http://qualificationswales.org/regulation/approved-and-designated-qualifications/as-a-level-approval-criteria-july-2016/?lang=en
[3] http://qualificationswales.org/regulation/monitoring-awarding-bodies/?lang=en&
[4] http://gov.wales/docs/dcells/publications/150727-fair-access-by-design-en.pdf

Subject aims and objectives

1. AS and A Level specifications in Further Mathematics must enable learners to:

   1.1. develop their understanding of mathematics and mathematical processes;

   1.2. develop their ability to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs;

   1.3. extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems;

   1.4. develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected;

   1.5. recognise how a situation may be represented mathematically and understand the relationship between 'real world' problems and mathematical models and how these can be refined and improved;

   1.6. use mathematics as an effective means of communication;

   1.7. read and comprehend mathematical arguments and articles concerning applications of mathematics;

   1.8. acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations;

   1.9. develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general;

   1.10. take increasing responsibility for their own learning and the evaluation of their own mathematical development.

Overarching themes

2. AS and A Level specifications in Further Mathematics must require learners to demonstrate the following overarching knowledge and skills. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out in *Appendix A*. The knowledge and skills required for AS Further Mathematics are shown in bold text within square brackets.

2.1. **Mathematical argument, language and proof**

Knowledge/Skill
[Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable]
[Understand and use mathematical language and syntax as set out in the content]
[Understand and use language and symbols associated with set theory, as set out in the content]
Understand and use the definition of a function; domain and range of functions
[Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics]

2.2. **Mathematical problem solving**

Knowledge/Skill
[Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved]
[Construct extended arguments to solve problems presented in an unstructured form, including problems in context]
[Interpret and communicate solutions in the context of the original problem]
[Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle]
[Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems]

2.3. **Mathematical modelling**

Knowledge/Skill
[Translate a situation in context into a mathematical model, making simplifying assumptions]
[Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student)]
[Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student)]

[Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate]
[Understand and use modelling assumptions]

AS and A Level Further Mathematics specifications must use the mathematical notation set out in *Appendix B* and must require learners to recall the mathematical formulae and identities set out in *Appendix C*.

Subject content

3. AS and A Level Mathematics specifications must reflect the subject aims and objectives in the content.

4. AS and A Level Further Mathematics specifications must build on the skills, knowledge and understanding set out in the GCSE subject content for GCSE Mathematics, GCSE Mathematics – Numeracy, and AS and A Level Mathematics.

5. AS and A Level Further Mathematics specifications must follow the prescribed core content outlined in *Appendix A* which will comprise approximately 50% of the overall A Level qualification content.

6. At least 30% of the content of the AS Further Mathematics specifications must be taken from the prescribed core content outlined in *Appendix A*.

7. AS Further Mathematics specifications must allow for AS Mathematics and AS Further Mathematics to be taught alongside each other.

   <u>Rationale Required:</u> On submitting the AS and A Level specification to Qualifications Wales for approval, the awarding body is required to provide rationale for:

   - *how the prescribed content is structured across AS and A2; and*
   - *the content that is not prescribed.*

Use of technology

8. The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A Level Further Mathematics.

   Calculators used must include the following features:

   8.1. an iterative function;

   8.2. the ability to compute summary statistics and access probabilities from standard statistical distributions.

Assessment objectives

9. The assessment objectives of the knowledge, understanding and skills required in the specification must target the following assessment objectives in line with the indicated weightings:

Objective	Requirements	Weighting		
		AS	**A2**	**A Level**
AO1	**Use and apply standard techniques** Learners should be able to: • select and correctly carry out routine procedures; and • accurately recall facts, terminology and definitions	50% (±5%)	50% (±5%)	50% (±5%)
AO2	**Reason, interpret and communicate mathematically** Learners should be able to: • construct rigorous mathematical arguments (including proofs); • make deductions and inference; • assess the validity of mathematical arguments; • explain their reasoning; and • use mathematical language and notation correctly. *Where questions/tasks targeting this assessment objective will also credit learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and in other contexts' (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).*	At least 10%	At least 15%	At least 15%
AO3	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical and non-mathematical contexts into mathematical processes; • interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations;	At least 10%	At least 15%	At least 15%

	• translate situations in context into mathematical models; • use mathematical models; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them. *Where questions/tasks targeting this assessment objective will also credit learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).*			

Scheme of assessment

10. AS and A Level Further Mathematics will be formally assessed through examinations only.

11. The use of scientific calculators will be permitted in assessments.

    <u>Rationale Required</u>: On submitting the AS and A Level specification to Qualifications Wales for approval, the awarding body is required to provide rationale for which types of calculators will be permitted for any given assessment component, together with any restrictions on functionality.

Appendix A – AS and A Level Further Mathematics

1. AS and A Level Further Mathematics specifications must include the following content. This, assessed in the context of the overarching themes, makes up approximately 50% of the total content of the full A Level.
2. Content required for AS Further Mathematics is shown in bold text within square brackets.

A. Proof

	Content
A1	Construct proofs using mathematical induction; contexts include sums of series, divisibility, and powers of matrices

B. Complex numbers

	Content
B1	**[Solve any quadratic equation with real coefficients; solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics)]**
B2	**[Add, subtract, multiply and divide complex numbers in the form $x + iy$ with x and y real; understand and use the terms 'real part' and 'imaginary part']**
B3	**[Understand and use the complex conjugate; know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs]**
B4	**[Use and interpret Argand diagrams]**
B5	**[Convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed)]**
B6	**[Multiply and divide complex numbers in modulus-argument form (knowledge of radians and compound angle formulae is assumed)]**
B7	**[Construct and interpret simple loci in the Argand diagram such $\|z - a\| > r$ and $\arg(z - a) = \theta$ (knowledge of radians is assumed)]**
B8	Understand de Moivre's theorem and use it to find multiple angle formulae and sums of series

B9	Know and use the definition $e^{i\theta} = \cos\theta + \mathrm{i}\sin\theta$ and the form $z = re^{i\theta}$
B10	Find the n distinct *n*th roots of $re^{i\theta}$ for $r \neq 0$ and know that they form the vertices of a regular n-gon in the Argand diagram
B11	Use complex roots of unity to solve geometric problems

C. Matrices

	Content
C1	**[Add, subtract and multiply conformable matrices; multiply a matrix by a scalar]**
C2	**[Understand and use zero and identity matrices]**
C3	**[Use matrices to represent linear transformations in 2-D; successive transformations; single transformations in 3-D (3-D transformations confined to reflection in one of $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes) (knowledge of 3-D vectors is assumed)]**
C4	**[Find invariant points and lines for a linear transformation]**
C5	**[Calculate determinants of 2 x 2]** and 3 x 3 matrices and interpret as scale factors, including the effect on orientation
C6	**[Understand and use singular and non-singular matrices; properties of inverse matrices]** **[Calculate and use the inverse of non-singular 2 x 2 matrices]** and 3 x 3 matrices
C7	Solve three linear simultaneous equations in three variables by use of the inverse matrix
C8	Interpret geometrically the solution and failure of solution of three simultaneous linear equations

D. Further algebra and functions

	Content
D1	**[Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations]**
D2	**[Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree)]**
D3	Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series
D4	Understand and use the method of differences for summation of series including use of partial fractions
D5	Find the Maclaurin series of a function including the general term
D6	Recognise and use the Maclaurin series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$, and be aware of the range of values of x for which they are valid (proof not required)

E. Further calculus

	Content
E1	Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity
E2	Derive formulae for and calculate volumes of revolution
E3	Understand and evaluate the mean value of a function
E4	Integrate using partial fractions (extend to quadratic factors $ax^2 + c$ in the denominator)
E5	Differentiate inverse trigonometric functions
E6	Integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and be able to choose trigonometric substitutions to integrate associated functions

F. Further vectors

	Content
F1	Understand and use the vector and Cartesian forms of an equation of a straight line in 3D
F2	Understand and use the vector and Cartesian forms of the equation of a plane
F3	Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane
F4	Check whether vectors are perpendicular by using the scalar product
F5	Find the intersection of a line and a plane Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane

G. Polar coordinates

	Content
G1	Understand and use polar coordinates and be able to convert between polar and cartesian coordinates
G2	Sketch curves with r given as a function of θ, including use of trigonometric functions
G3	Find the area enclosed by a polar curve

H. Hyperbolic functions

	Content
H1	Understand the definitions of hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$ including their domains and ranges, and be able to sketch their graphs
H2	Differentiate and integrate hyperbolic functions
H3	Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges
H4	Derive and use the logarithmic forms of the inverse hyperbolic functions

H5	Integrate functions of the form $(x^2 + a^2)^{-\frac{1}{2}}$ and $(x^2 - a^2)^{-\frac{1}{2}}$ and be able to choose substitutions to integrate associated functions

I. Differential equations

	Content
I1	Find and use an integrating factor to solve differential equations of form $\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x)$ and recognise when it is appropriate to do so
I2	Find both general and particular solutions to differential equations
I3	Use differential equations in modelling in a variety of contexts
I4	Solve differential equations of form $y'' + ay' + by = 0$ where a and b are constants by using the auxiliary equation
I5	Solve differential equations of form $y'' + ay' + by = f(x)$where a and b are constants by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $f(x)$ is a polynomial, exponential or trigonometric function)
I6	Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation

Appendix B: mathematical notation for AS and A Level Further Mathematics

The tables below set out the notation that must be used by AS and A Level Further Mathematics specifications. Learners will be expected to understand this notation without the need for further explanation.

AS learner will be expected to understand notation that relates to AS content, and will not be expected to understand notation that relates only to A Level content.

1	**Set Notation**	
1.1	$\in$	Is an element of
1.2	$\notin$	Is not an element of
1.3	$\subseteq$	Is a subset of
1.4	$\subset$	Is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	The set with elements $x_1, x_2, \ldots$
1.6	$\{x: \ldots\}$	The set of all x such that …
1.7	$\mathrm{n}(A)$	The number of elements in set A
1.8	$\emptyset$	The empty set
1.9	$\boldsymbol{\varepsilon}$	The universal set
1.10	A'	The complement of the set A
1.11	$\mathbb{N}$	The set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	The set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
1.13	$\mathbb{Z}^+$	The set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	The set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	The set of real numbers
1.16	$\mathbb{Q}$	The set of rational numbers $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	Union
1.18	$\cap$	Intersection
1.19	(x, y)	The ordered pair x, y
1.20	$[a, b]$	The closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
1.21	$[a, b)$	The interval $\{x \in \mathbb{R}: a \leq x < b\}$
1.22	$(a, b]$	The interval $\{x \in \mathbb{R}: a < x \leq b\}$
1.23	(a, b)	The open interval $\{x \in \mathbb{R}: a < x < b\}$
1.24	$\mathbb{C}$	The set of complex numbers
2	**Miscellaneous Symbols**	
2.1	$=$	Is equal to
2.2	$\neq$	Is not equal to
2.3	$\equiv$	Is identical to or congruent to
2.4	$\approx$	Is approximately equal to
2.5	∞	Infinity
2.6	$\propto$	Is proportional to
2.7	$\therefore$	Therefore
2.8	$\because$	Because
2.9	$<$	Is less than

2.10	$\leqslant, \leq$	Is less than or equal to, is not greater than
2.11	$>$	Is greater than
2.12	$\geqslant, \geq$	Is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p *to* q)
2.16	a	First term for an arithmetic or geometric sequence
2.17	l	Last term for arithmetic sequence
2.18	d	Common difference for an arithmetic sequence
2.19	r	Common ratio for a geometric sequence
2.20	S_n	Sum to n terms of a sequence
2.21	S_∞	Sum to infinity of a sequence
3	**Operations**	
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b
3.3	$a \times b$, $a\ b$, $a.b$	a multiplied by b
3.4	$a \div b, \frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times, a_2 \times, \ldots \times\ a_n$
3.7	$\sqrt{a}$	The non-negative square root of a
3.8	$\lvert a \rvert$	The modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1$, $n \in \mathbb{N}; 0! = 1$
3.10	$\binom{n}{r}, {}^{n}C_r\ {}_{n}C_r$	The binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leqslant n$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$
4	**Functions**	
4.1	$\mathrm{f}(x)$	The value of the function f at x
4.2	$\mathrm{f} : x \mapsto y$	The function f maps the element x to the element y
4.3	f^{-1}	The inverse function of the function f
4.4	gf	The composite function of f and g which is defined by gf(x) = g(f(x))
4.5	$\lim_{x \to a} \mathrm{f}(x)$	The limit of f(x) as x tends to a
4.6	$\Delta x, \delta x$	An increment of x

4.7	$\frac{\mathrm{d}y}{\mathrm{d}x}$	The derivative of y with respect to x
4.8	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	The n th derivative of y with respect to x
4.9	$\mathrm{f}'(x)$, $\mathrm{f}''(x)$,…, $\mathrm{f}^n(x)$	The first, second …, n^{th} derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x}$, $\ddot{x}$, …	The first, second, … derivatives of x with respect to t
4.11	$\int y\ \mathrm{d}x$	The indefinite integral of y with respect to x
4.12	$\int_a^b y\ \mathrm{d}x$	The definite integral of y with respect to x between the limits $x = a$ and $x = b$
5	**Exponential and Logarithmic Functions**	
05.1	e	Base of natural logarithms
5.2	e^x, $\exp x$	Exponential function of x
5.3	$\log_a x$	Logarithm to the base a of x
5.4	$\ln x$, $\log_e x$	Natural logarithm of x
6	**Trigonometric Functions**	
6.1	$\left.\begin{matrix}\sin, \cos, \tan,\\ \mathrm{cosec}, \sec, \cot\end{matrix}\right\}$	The trigonometric functions
6.2	$\left.\begin{matrix}\sin^{-1}, \cos^{-1}, \tan^{-1}\\ \arcsin, \arccos, \arctan\end{matrix}\right\}$	The inverse trigonometric functions
6.3	°	Degrees
6.4	rad	Radians
6.5	$\left.\begin{matrix}\mathrm{cosec}^{-1}, \sec^{-1}, \cot^{-1}\\ \mathrm{arccosec}, \mathrm{arcsec}, \mathrm{arccot}\end{matrix}\right\}$	The inverse trigonometric functions
6.6	$\left.\begin{matrix}\sinh, \cosh, \tanh,\\ \mathrm{cosech}, \mathrm{sech}, \coth\end{matrix}\right\}$	The hyperbolic functions
6.7	$\left.\begin{matrix}\sinh^{-1}, \cosh^{-1}, \tanh^{-1},\\ \mathrm{cosech}^{-1}, \mathrm{sech}^{-1}, \coth^{-1}\end{matrix}\right\}$ $\left.\begin{matrix}\mathrm{arsinh}, \mathrm{arcosh}, \mathrm{artanh},\\ \mathrm{arcosech}, \mathrm{arsech}, \mathrm{arcoth}\end{matrix}\right\}$	The inverse hyperbolic functions
7	**Complex Numbers**	
7.1	i , j	Square root of -1
7.2	$x + \mathrm{i}y$	Complex number with real part x and imaginary part y
7.3	$r(\cos\theta + \mathrm{i}\sin\theta)$	Modulus argument form of a complex number with modulus r and argument θ
7.4	z	A complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$
7.5	$\mathrm{Re}(z)$	The real part of z, $\mathrm{Re}(z) = x$
7.6	$\mathrm{Im}(z)$	The imaginary part of z, $\mathrm{Im}(z)=y$
7.7	$\lvert z\rvert$	The modulus of z, $\lvert z\rvert = \sqrt{x^2 + y^2}$
7.8	$\arg(z)$	The argument of z, $\arg(z) = \theta$, $-\pi < \theta \leq \pi$
7.9	z^*	The complex conjugate of z, $x - iy$

8	**Matrices**	
8.1	$\mathbf{M}$	A matrix $\mathbf{M}$
8.2	$\mathbf{0}$	Zero matrix
8.3	$\mathbf{I}$	Identity matrix
8.4	$\mathbf{M}^{-1}$	The inverse of the matrix $\mathbf{M}$
8.5	$\mathbf{M}^{\mathrm{T}}$	The transpose of the matrix $\mathbf{M}$
8.6	$\Delta, \det \mathbf{M}$ *or* $\lvert\mathbf{M}\rvert$	The determinant of the square matrix $\mathbf{M}$
8.7	**Mr**	Image of column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$
9	**Vectors**	
9.1	$\mathbf{a}$, $\underline{\mathrm{a}}$, $\underset{\sim}{\mathrm{a}}$	The vector $\mathbf{a}$, $\underline{\mathrm{a}}$, $\underset{\sim}{\mathrm{a}}$;these alternatives apply throughout section 9
9.2	$\overrightarrow{\mathrm{AB}}$	The vector represented in magnitude and direction by the directed line segment AB
9.3	$\hat{\mathbf{a}}$	A unit vector in the direction of $\mathbf{a}$
9.4	$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	Unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert\mathbf{a}\rvert$, a	The magnitude of $\mathbf{a}$
9.6	$\lvert\overrightarrow{\mathrm{AB}}\rvert$, AB	The magnitude of $\overrightarrow{\mathrm{AB}}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}$, $a\mathbf{i} + b\mathbf{j}$	Column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	Position vector
9.9	$\mathbf{s}$	Displacement vector
9.10	$\mathbf{v}$	Velocity vector
9.11	$\mathbf{a}$	Acceleration vector
9.12	$\mathbf{a}.\mathbf{b}$	The scalar product of $\mathbf{a}$ and $\mathbf{b}$
10	**Differential Equations**	
10.1	ω	Angular speed
11	**Probability and Statistics**	
11.1	A, B, C, etc.	Events
11.2	$A \cup B$	Union of the events A and B
11.3	$A \cap B$	Intersection of the events A and B
11.4	$\mathrm{P}(A)$	Probability of the event A
11.5	A'	Complement of the event A
11.6	$\mathrm{P}(A\lvert B)$	Probability of the event A conditional on the event B
11.7	X, Y, R, etc.	Random variables
11.8	x, y, r, etc.	Values of the random variables X, Y, R etc.
11.9	x_1,x_2, …	Values of observations
11.10	f_1,f_2, …	Frequencies with which the observations x_1,x_2, … occur
11.11	$\mathrm{p}(x)$, $\mathrm{P}(X=x)$	Probability function of the discrete random variable X
11.12	p_1,p_2, …	Probabilities of the values x_1,x_2, … of the discrete random variable X

11.13	$\mathrm{E}(X)$	Expectation of the random variable X
11.14	$\mathrm{Var}(X)$	Variance of the random variable X
11.15	~	Has the distribution
11.16	$\mathrm{B}(n, p)$	Binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	q = 1-p for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	$\mathrm{Z} \sim \mathrm{N}(0,1)$	Standard Normal distribution
11.20	ϕ	Probability density function of the standardised Normal variable with distribution $\mathrm{N}(0,1)$
11.21	Φ	Corresponding cumulative distribution function
11.22	μ	Population mean
11.23	σ^2	Population variance
11.24	σ	Population standard deviation
11.25	$\overline{x}$	Sample mean
11.26	s^2	Sample variance
11.27	s	Sample standard deviation
11.28	H_0	Null hypothesis
11.29	H_1	Alternative hypothesis
11.30	r	Product moment correlation coefficient for a sample
11.31	ρ	Product moment correlation coefficient for a population
12	**Mechanics**	
12.1	kg	Kilograms
12.2	m	Metres
12.3	km	Kilometres
12.4	m/s, $\mathrm{m\ s^{-1}}$	Metres per second (velocity)
12.5	$\mathrm{m/s^2}$, $\mathrm{m\ s^{-2}}$	Metres per second per second (acceleration)
12.6	F	Force or resultant force
12.7	N	Newton
12.8	N m	Newton metre (moment of a force)
12.9	t	Time
12.10	s	Displacement
12.11	u	Initial velocity
12.12	v	Velocity or final velocity
12.13	a	Acceleration
12.14	g	Acceleration due to gravity
12.15	μ	Coefficient or friction

Appendix C - mathematical formulae and identities

Learners must be able to use the following formulae and identities for AS and A Level Further Mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure Mathematics

Quadratic Equations

$ax^2+bx+c=0$ has roots $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Laws of Indices

$a^x a^y \equiv a^{x+y}$

$a^x \div a^y \equiv a^{x-y}$

$(a^x)^y \equiv a^{xy}$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x \ for\ a > 0\ and\ x > 0$

$\log_a x + \log_a y \equiv \log_a(xy)$

$\log_a x - \log_a y \equiv \log_a\left(\frac{x}{y}\right)$

$k\log_a x \equiv \log_a(x^k)$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1,y_1) has equation

$y-y_1=m(x-x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1m_2 = -1$

Sequences

General term of an arithmetic progression:

$u_n = a + (n-1)d$

General term of a geometric progression:

$u_n = ar^{n-1}$

Trigonometry

In the triangle ABC

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area = $\frac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and Area of circle, radius r and diameter d :

$C = 2\pi r = \pi d \quad A = \pi r^2$

Pythagoras' Theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse:

$c^2 = a^2 + b^2$

Area of trapezium = $\frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A:

$s = r\theta \quad A = \frac{1}{2}r^2\theta$

Complex Numbers

For two complex numbers $z_1 = z_1 e^{i\theta_1}$ and $z_2 = z_2 e^{i\theta_2}$:

$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$

Loci in the Argand diagram:

$|z-a| = r$ is a circle radius r centred at a

$\arg(z-a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis

Exponential Form:

$e^{i\theta} = \cos\theta + i\sin\theta$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

The inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix **AB** is the transformation represented by matrix **B** followed by the transformation represented by matrix **A**.

For matrics **A, B**:

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Algebra

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

For $ax^2 + bx + c = 0$ with roots α and β:

$\alpha + \beta = \frac{-b}{a}$ $\alpha\beta = \frac{c}{a}$

For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ:

$\sum \alpha = \frac{-b}{a}$ $\sum \alpha\beta = \frac{c}{a}$ $\sum \alpha\beta\gamma = \frac{-d}{a}$

Hyperbolic Functions

$$\cosh x \equiv \frac{1}{2}(e^{x} + e^{-x})$$

$$\sinh x \equiv \frac{1}{2}(e^{x} - e^{-x})$$

$$\tanh x \equiv \frac{\sinh x}{\cosh x}$$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
$\sinh kx$	$k\cosh kx$
$\cosh kx$	$k\sinh kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
$\cosh kx$	$\frac{1}{k}\sinh kx + c$
$\sinh kx$	$\frac{1}{k}\cosh kx + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f(g(x)) + c$

Area under a curve $= \int_a^b y\,dx\ (y \geq 0)$

Volumes of revolution about x and y axes:

$V_x = \pi \int_a^b y^2 \mathrm{d}x$
$V_y = \pi \int_c^d x^2 dy$

Simple Harmonic Motion:

$\ddot{x} = -\omega^2 x$

Vectors

$|x\mathbf{i} + y\mathbf{j} + \mathrm{z}\mathbf{k}| = \sqrt{(x^2 + y^2 + z^2)}$

Scalar product of two vectors $\mathbf{a}=\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |a||b|\cos\theta$$

where θ is the acute angle between the vectors $\mathbf{a}$ and $\mathbf{b}$

The equation of the line through the point with position vector a parallel to vector b is:

$\mathbf{r} = \mathbf{a} + t\mathbf{b}$

The equation of the plane containing the point with position vector $\mathbf{a}$ and perpendicular to vector $\mathbf{n}$ is:

$(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$

Mechanics

Forces and Equilibrium

Weight = mass × g

Friction $F \leq \mu R$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration:

$v = \frac{\mathrm{d}r}{\mathrm{d}t}$ $\quad a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2r}{\mathrm{d}t^2}$

$r = \int v\mathrm{d}t$ $\quad v = \int a\ \mathrm{d}t$

Statistics

The mean of a set of data $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

The standard Normal variable: $Z = \frac{X-\mu}{\sigma}$ where $X \sim N(\mu, \sigma^2)$

Further information

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